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Axially Loaded Column Subjected to Lateral Pressure

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The buckling behavior of columns subjected to axial load and lateral pressure is of general interest in aerospace applications and in the industrial manufacture of extrusions. A solution to this problem may be derived rather easily and, surprisingly, it is not generally known. Consider the column shown in the sketch at the top of Fig. 1; it is subjected to an end load pA and a lateral pressure q. The cross-sectional shape of the column is shown circular but it may be any shape that is constant with the coordinate x including closed thin-wall tubular shapes. The coordinate z is taken normal to the axis of the least moment of inertia of the cross section of the column.

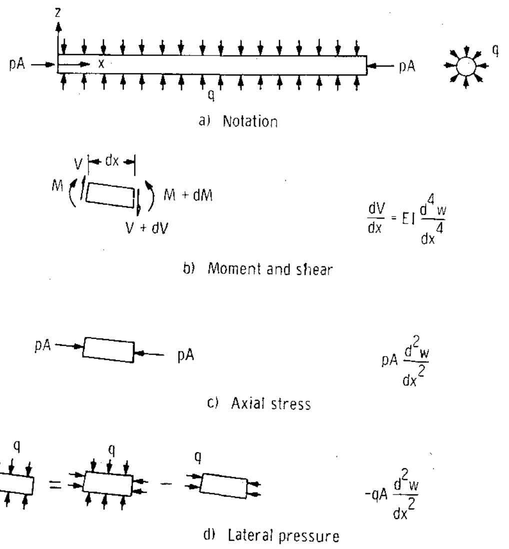

Fig. 1 Lateral forces entering into the equilibrium of an element of a buckled column

A differential equation for the lateral deflection w of the column may be obtained from considerations of the equilibrium of an element of the buckled column. For equilibrium in the direction of z, the following equation is obtained as the sum of the terms listed on the right-hand side of Fig. 1b–1d.

$$EI\,(d^4w/dx^4) + (p - q)A(d^2w/dx^2) = 0 \qquad (1)$$

The qA term (Fig. 1d) is obtained without the necessity of integrating over the surface of the column element by making use of the fact that any body subjected to hydrostatic pressure is in equilibrium.

Solution of Eq. (1) leads to the eigenvalue

$$(p - q) = p_0 \qquad (2)$$

or to

$$(p/p_0) - (q/p_0) = 1 \qquad (3)$$

where p_0A, which is a function of the boundary conditions at the ends of the column, is the compressive axial load required to buckle the column in the absence of lateral pressure, and A is the cross-sectional area of solid-section columns and is the area enclosed by the outside dimensions of the walls of tubular columns. Equation (3), which gives the interaction between axial and lateral pressure on column buckling and which is independent of boundary conditions, is plotted in Fig. 2. It can be seen from Fig. 2 that a column in a hydrostatic pressure field lies in the stable region and requires an additional end load equal to p_0A in order to reach the stability boundary; the hydrostatic pressure may be a tensile or negative pressure and, in the case of a closed tubular strut, may be produced by internal pressure. Hence a tubular strut subjected to tensile stresses of pressurization and to a compressive axial load equal to the column buckling load of the unpressurized tube will buckle even though the tensile stresses of pressurization may be greater than the compressive stresses from the axial load; that is, it is possible for a tube to buckle as a column while subjected to tensile stresses in both the circumferential and axial directions. The pressures p and q of Eq. (3) and Fig. 2 are pressures applied to the outside of the column. Hence, for tubular columns where the area A enclosed by the outside dimensions of the tube differs appreciably from the area A_i enclosed by the inside dimensions of the tube and where a hydrostatic pressure field is achieved by internal pressurization, a correction of A_i/A must be applied to the internal pressure in order to obtain the equivalent external pressure required in Eq. (3) and Fig. 2.

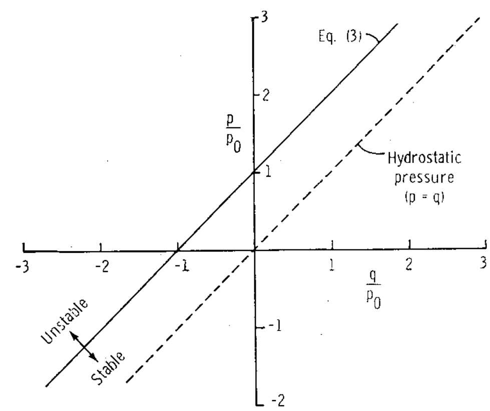

Fig. 2 Interaction curve for column subjected to axial load and lateral pressure

Solutions to the stability of pressurized circular thin-wall columns subjected to the particular loading case $p = 0$ and $q = -p_0$ are given in Refs. 1 and 2. The stability of rectangular solid sections is discussed in Ref. 3, but erroneous conclusions are drawn because of an incorrect sign on the pressure q in the equation of Ref. 3 corresponding to Eq. (2) of the present note.

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